Add Motion Sensing to Your Device

ESC-320

60 minute session

Wednesday, 4 May 2011

3:15 pm to 4:15 pm

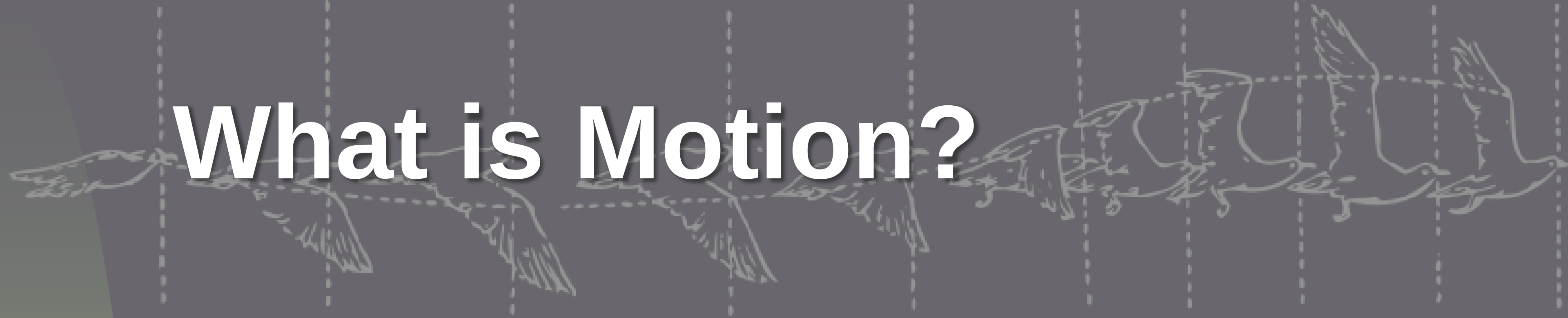
Matthew Liberty

President

Jetperch LLC

Agenda

- What is motion?
- Frames of reference
- Available sensors
- Using sensors
- Determine the motion for your application
- Conclusion & Questions



What is Motion?

- If you don’t know what motion is, then how did you get here?

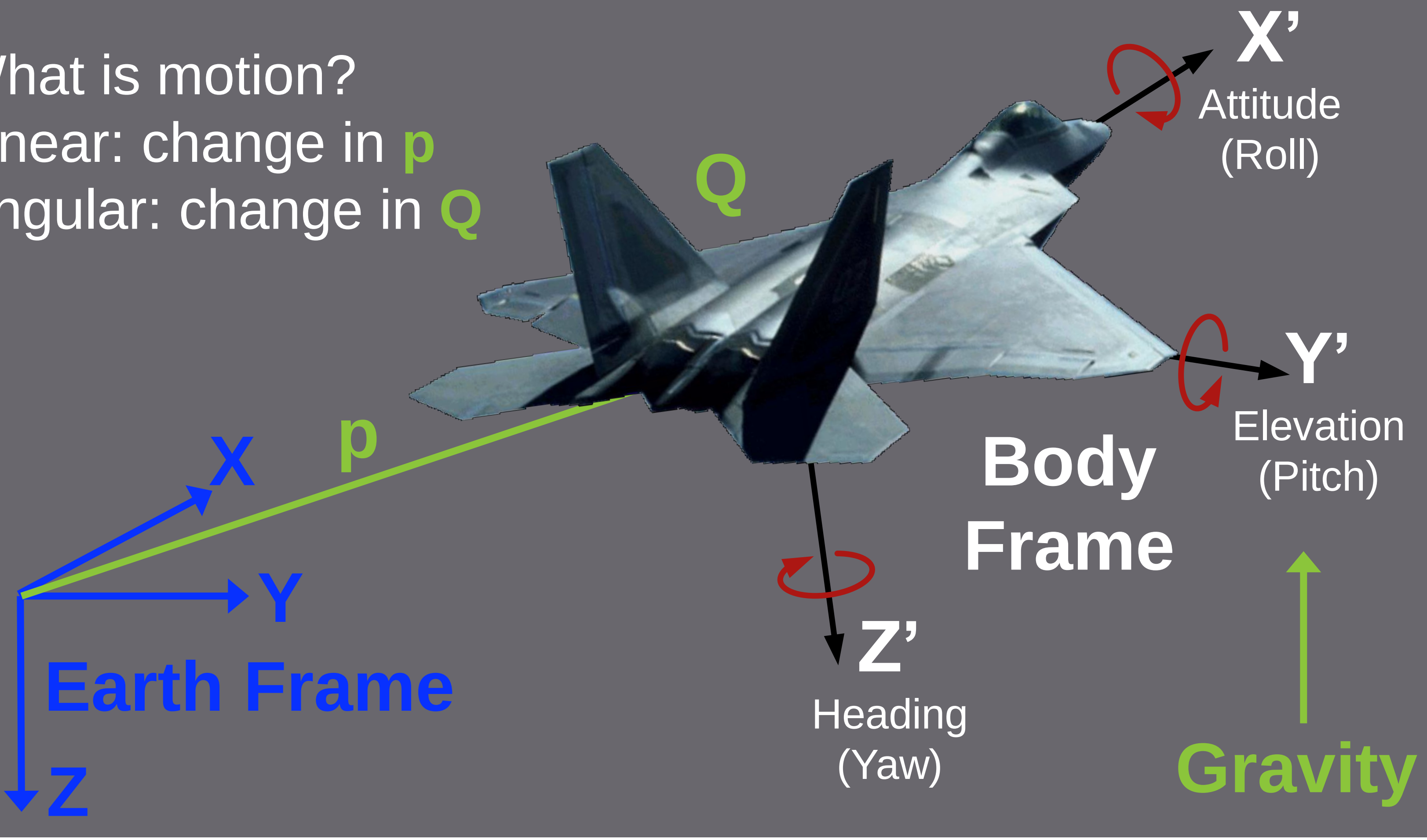
- From Merriam-Webster’s: an act, process, or instance of changing place : movement
- Three linear degrees of freedom: XYZ
- Three angular degrees of freedom:
  yaw, pitch, roll

Why Add Motion Sensing?

- Motion is natural
  - Primary method of communication that predates speech
  - Enables simpler interaction design and increase ease of use
- Technology is becoming ubiquitous – now present in most mobile phones
- Low cost

Frames of Reference

What is motion?
Linear: change in **p**
Angular: change in **Q**



Basic Motion Equations

- Given 3D linear position ***p***
  - Velocity $\boldsymbol{v} = \frac{d}{dt}\boldsymbol{p} = \dot{\boldsymbol{p}}$
  - Acceleration $\boldsymbol{a} = \frac{d}{dt}\boldsymbol{v} = \dot{\boldsymbol{v}} = \ddot{\boldsymbol{p}}$
- Given 1D angular position θ
  - Velocity $\omega = \frac{d}{dt}\theta = \dot{\theta}$
  - Acceleration $\dot{\omega} = \frac{d}{dt}\omega = \ddot{\theta}$

Angular Motion

- The 3D representation requires rotations
  - A rotation is a mathematical transformation that maintains length and distance
  - Rotations are non-commutative
- Coordinate system transform and vector rotation are equal and opposite
  - Easy to confuse
  - Tends to cause annoying, hard to find bugs
- Calculus is non-linear

Euler Angles

- Three angles that represent successive rotation operations around different axes
- 12 such sequences exist
  - For example Z (heading) Y (altitude) X (roll)
- Euler angles are numerically unstable due to gimbal lock
- Good for conceptualization, not calculation
- Do not use Euler angles for your application!

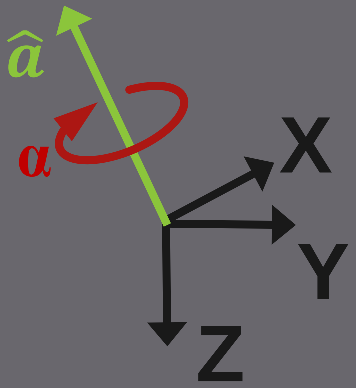
Perturbation

- Three angles representing an incremental displacement about the body-frame coordinate axes
- Not good for representing angular position
- Great for integrating angular velocity with a small angle approximation: $\omega \Delta t$
- Can often simplify real-world calculations

Direction Cosine Matrix

- Has 9 values instead of the 3 in Euler angles!
- 3x3 matrix with constraints
  - Determinant is 1
  - The inverse equals the transpose: $\boldsymbol{Q}^{-1} = \boldsymbol{Q}^{T}$
  - $\boldsymbol{\Omega} = \boldsymbol{Q}^{T}\,\dot{\boldsymbol{Q}} = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$
  - $\boldsymbol{Q}(n) = (\boldsymbol{I} + \boldsymbol{\Omega}\,\Delta t)\boldsymbol{Q}(n-1)$ [Euler integration]

Quaternion
(similar to axis-angle)



- Length 4 vector with special algebra
  - Analogous to complex numbers
  - $\boldsymbol{q} = \langle q_0, \ \boldsymbol{v} \rangle = (q_0, \ q_x, \ q_y, \ q_z)$
  - $\boldsymbol{p}\,\boldsymbol{q} = \langle a\,b - \boldsymbol{v} \cdot \boldsymbol{w}, \ \boldsymbol{v} \times \boldsymbol{w} + a\,\boldsymbol{w} + b\,\boldsymbol{v} \rangle$
- All magnitude 1 quaternions can be rotations
  - $\boldsymbol{q} = \langle \cos\frac{\alpha}{2}, \ \hat{\boldsymbol{a}}\,\sin\frac{\alpha}{2} \rangle$: note axis $\hat{\boldsymbol{a}}$, angle α
- Transform: $\langle 0, \ \boldsymbol{w} \rangle = \boldsymbol{q}^* \langle 0, \ \boldsymbol{v} \rangle\, \boldsymbol{q}$
- Rotation: $\langle 0, \ \boldsymbol{x} \rangle = \boldsymbol{q}\, \langle 0, \ \boldsymbol{v} \rangle\, \boldsymbol{q}^*$

Agenda

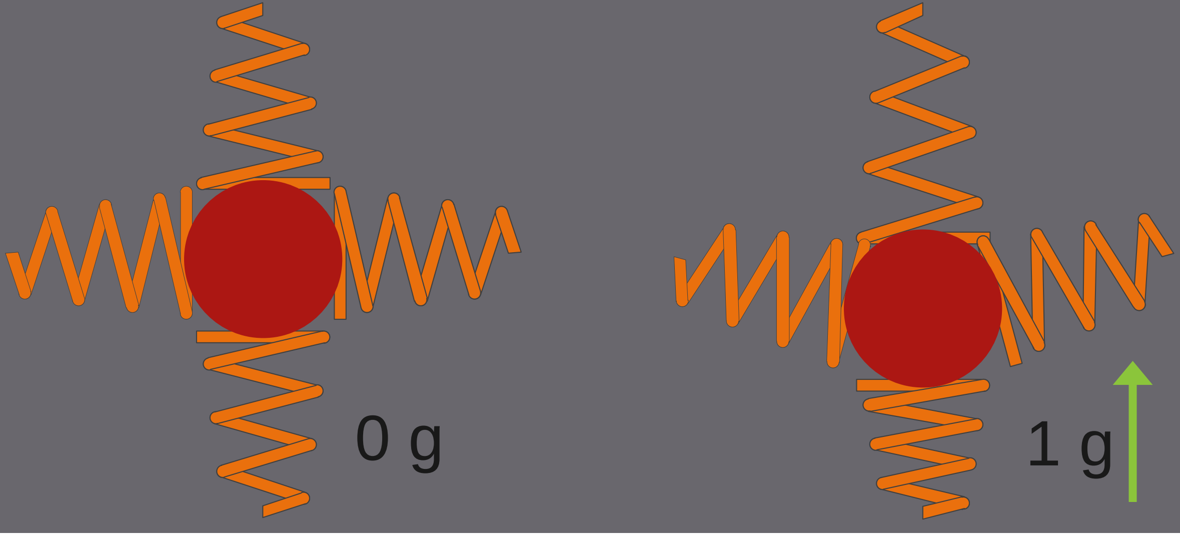
- What is motion?
- Frames of reference
- Available sensors
- Using sensors
- Determine the motion for your application
- Conclusion & Questions

Available Sensors

- Linear accelerometer
- Gyroscope
- Magnetometer
- Camera
- GPS
- and more!

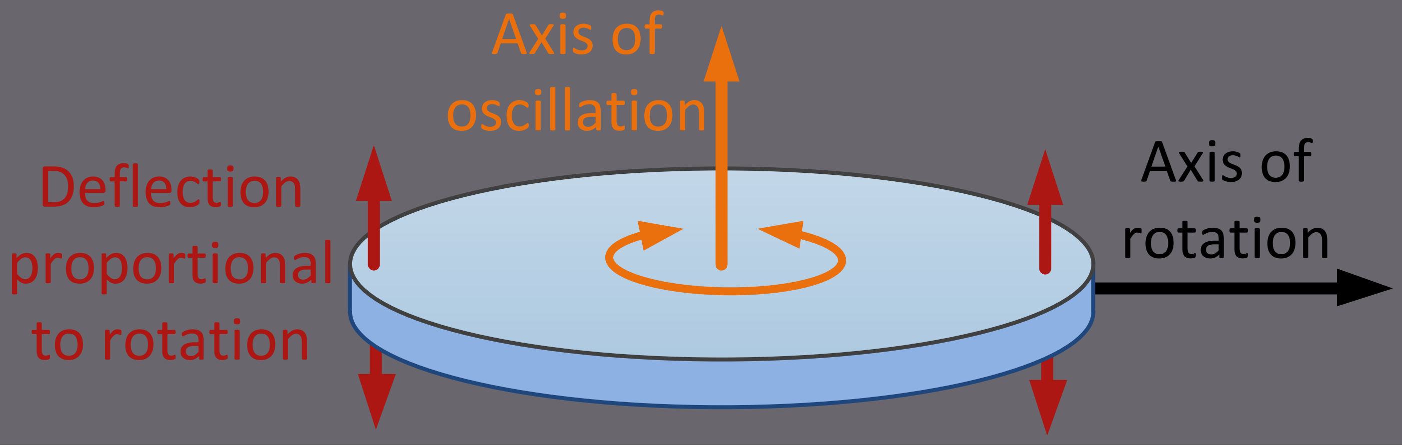
Linear Accelerometer

- Measures linear acceleration AND gravity
  - $\boldsymbol{a} = \boldsymbol{Q}\,(\ddot{\boldsymbol{p}} + \boldsymbol{g})$
- Mass on a spring



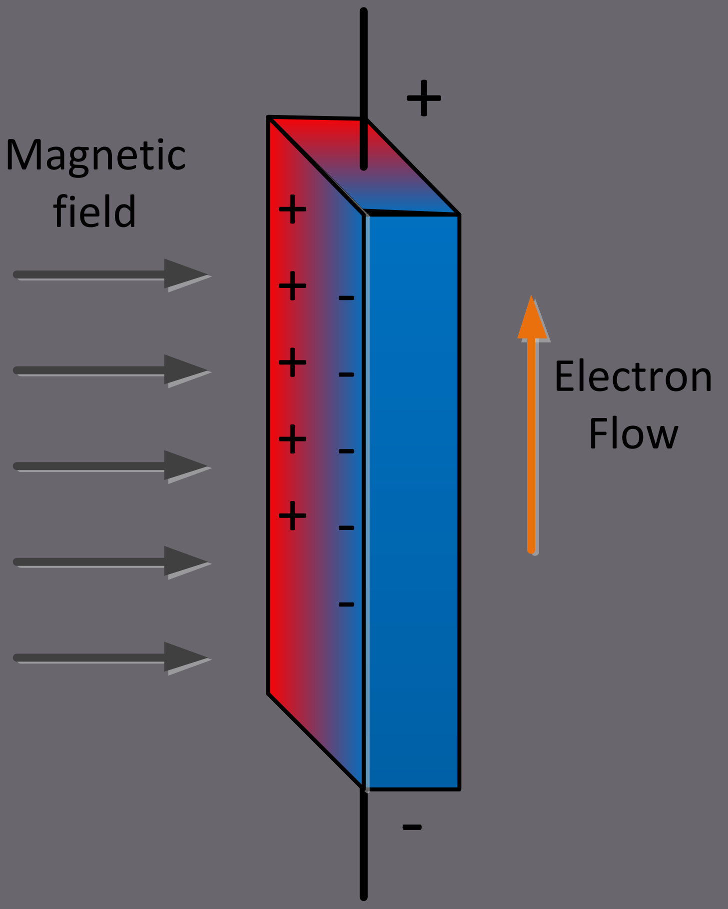
Gyroscope

- Measures angular velocity (Coriolis vibratory gyroscope)
- Uses an oscillating mass that is deflected by the Coriolis force perpendicular to both the rotation and the oscillation



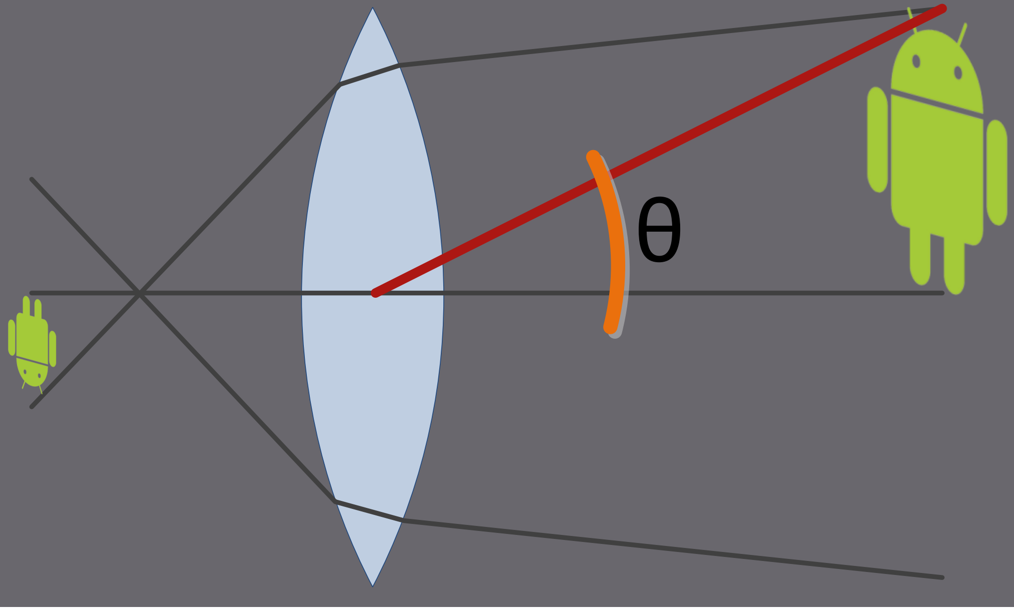
Magnetometer

- Measures the magnetic field
- Uses hall effect



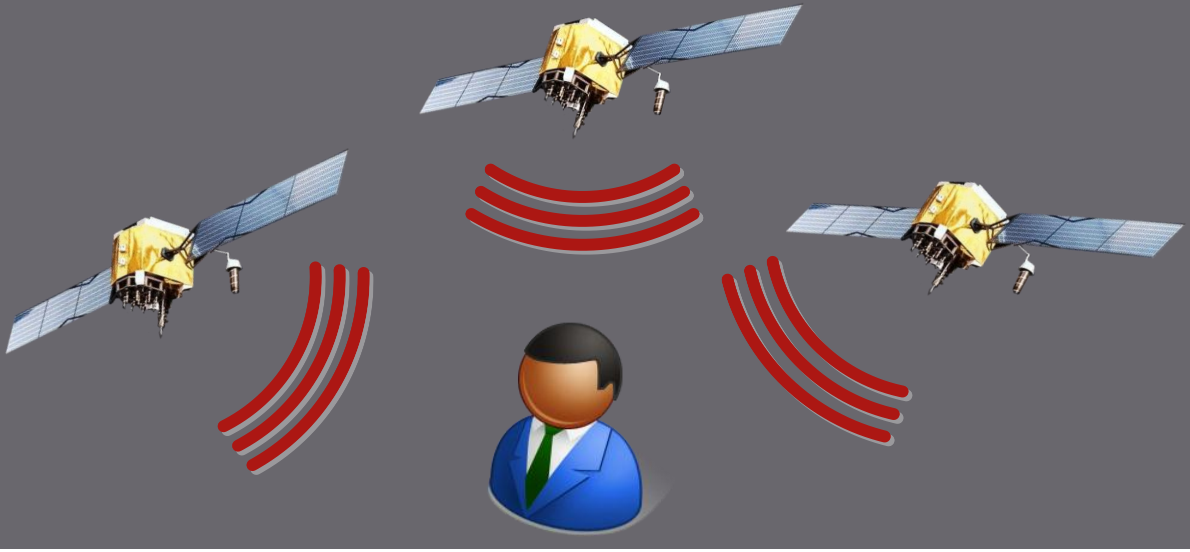
Camera

- Measures angular direction to object
- Can infer distance from size



GPS

- Measures linear position relative to Earth
- Uses synchronized signal arrival time from a constellation of satellites



Sensing Method Summary

Type	Measures	Advantages	Disadvantages
Accelerometer	Linear acceleration including gravity	Small & inexpensive Low power Long-term stable measurement of 2/3 angular position Can include orientation, wake-on-motion and tap detection	Double integration to linear position Difficult to separate gravity from linear acceleration
MEMS gyroscope (rate gyro)	Angular velocity	Responsive	Integration to angular position Higher power Bias Instability
Magnetometer	Magnetic field	Small & inexpensive Low power Long-term stable measurement of 2/3 angular position	Magnetic fields are subject to fringing Earth's magnetic field is weak and variable Other magnetic sources
Camera	Angular direction with distance approximation	Long-term stable	Low update rate Large data quantity Requires suitable lighting
GPS	Linear position	Long-term stable	May not work indoors Power hungry

Using Sensors

- Sensors typically have errors
  - Bias (Zero value offset)
  - Sensitivity accuracy
  - Sensitivity non-linearity
  - Cross-axis sensitivity (coupling)
  - Responsiveness to non-intended signals
- Other concerns
  - Noise, quantization and resolution
  - Dynamic range
  - Latency and bandwidth
- Must select sensors that meet your performance goals

Example: Accelerometer

- Measures linear acceleration AND gravity
- When located at **p**: $\boldsymbol{a} = \boldsymbol{Q}\,(\ddot{\boldsymbol{p}} + \boldsymbol{g})$
- When located on rigid body away from **p**
  - $\boldsymbol{a} = \boldsymbol{Q}(\ddot{\boldsymbol{p}} + \boldsymbol{g}) + \boldsymbol{Q}\,\ddot{\boldsymbol{Q}}^{T}\,\boldsymbol{s}$
  - $\boldsymbol{a} = \boldsymbol{Q}(\ddot{\boldsymbol{p}} + \boldsymbol{g}) + \dot{\boldsymbol{\omega}} \times \boldsymbol{s} + \boldsymbol{\omega} \times \boldsymbol{\omega} \times \boldsymbol{s}$
- Tilt accuracy depends upon error factors. Considering a 2D bias only case:

$$\theta_{error} = \tan^{-1}\frac{1000\ \sin\theta \pm b_x}{1000\ \cos\theta \pm b_z} - \theta$$

$$\theta_{error} = 8° \; when\; \theta = 45°, b = \ 100\, mg$$

Sensor Fusion

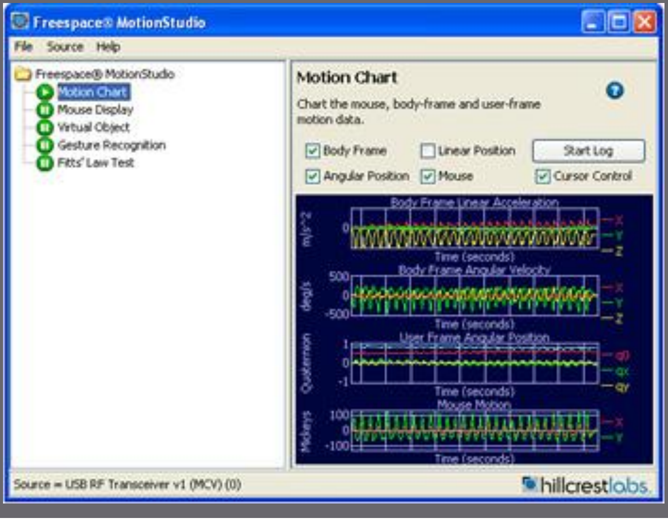
- Why choose 1 sensor when you can choose many!
- Use statistics to combine measurements

$$\frac{\sigma_A^2}{\sigma_A^2+\sigma_B^2} m_B + \frac{\sigma_B^2}{\sigma_A^2+\sigma_B^2} m_A \qquad , \qquad \frac{\sigma_A^2\,\sigma_B^2}{\sigma_A^2+\sigma_B^2}$$

- Kalman filtering is the starting point
  - Optimal method for linear systems with normally distributed process and measurement noise
  - Similar methods available for non-linear systems
- Off-the-shelf solutions are available

Motion Demonstration

- XYZ accelerometer
- XYZ rate gyroscope
- Sensor fusion for angular position



Motion Chart

Chart the mouse, body-frame and user-frame motion data.

☑ Body Frame ☐ Linear Position

☑ Angular Position ☑ Mouse ☑ Cursor Control

Source = USB RF Transceiver v1 (MCV) (0)

Agenda

- What is motion?
- Frames of reference
- Available sensors
- Using sensors
- Determine the motion for your application
- Conclusion & Questions

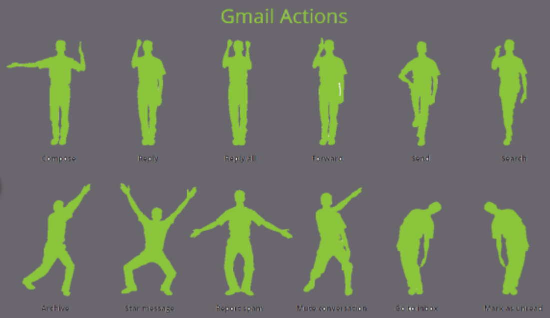
Example: Joystick

- Measure roll (x-axis) and pitch (y-axis)
- Use a ±2 g accelerometer
  - $\theta_x = \sin^{-1}\frac{x}{\sqrt{x^2+y^2+z^2}}$
  - $\theta_y = \sin^{-1}\frac{y}{\sqrt{x^2+y^2+z^2}}$
  - Any linear motion introduces error
- For improved performance, add a 2-axis gyro with sensor fusion

Example: Cursor control

- Simplest method: a ±500 °/s 2D gyro to measure z-axis and y-axis angular velocity
- Better method: add a 3D linear accelerometer for orientation compensation so that rotating the device “up” always moves the cursor up, regardless of how it is held
- Best method: use a ±2000 °/s 3D gyro for added stability

Example: Gestures



- Gesture: a motion that is recognized and translated into an event
- The required motion sensing depends upon the defined gesture set!
  - Gestures range from simple actions such as tapping to complex input, such as letters.
  - Must measure motion sufficiently to distinguish between the defined gestures across users
- For simple gestures, an accelerometer may be sufficient

Example: Golf Swing

- Simplest method: Use a 3D accelerometer to determine the start of the backswing, end of backswing and rough magnitude
- Better method: Add a ±2500 °/s 1D gyro to accurately measure the swing
- Best method: Add a ±2500 °/s 3D gyro to accurately measure hook/slice and out/in

Example: Navigation

- Long-term navigation is one of the most challenging applications of motion sensing
  - At minimum, measure linear position
  - Location based services wants angular position
  - Dead-reckoning based upon inertial sensing is problematic due to double integration & gravity
- Modern solutions combine XYZ accelerometer, XYZ gyro, XYZ magnetometer, barometer and GPS with Extended Kalman filtering

How to Add Motion Sensing

- Determine motion requirements
- Select sensor(s)
- Integrate at hardware level (I^2C, SPI)
  - Recommend digital sensors to simplify PCB and hardware design
- Integrate at software level (drivers)
- Implement motion processing algorithm(s)
- Develop application(s)

Conclusion

- Motion adds new options and opportunities for product design
- Amount of effort to add motion varies with the motion requirements
  - Can be simple as adding an accelerometer
  - Can be as complicated as dead-reckoning
- Easy to get started

Questions?

Add Motion Sensing to Your Device

ESC-320

Matthew Liberty

President

Jetperch LLC

www.jetperch.com | matt@jetperch.com